hence,

$$I \approx c_3c_4(f_o' - f_i)$$

and

$$f_o' \approx f_o + c_1c_3c_4(f_o' - f_o).$$

Solving for f_o' we have from (2) and (3)

$$\frac{\Delta\phi'}{\Delta\phi} \approx \frac{f_o' - f_i}{f_o - f_i} \approx \frac{1}{1 - c_1c_3c_4}. \tag{5}$$

For the YIG oscillators used $C_1 = 17$ MHz/mA. For the feedback system used $C_3 = -(360)/(2\pi) \times 8 \times 10^{-3}$ mA/rad.

Inserting values previously given into (4) we find $C_4 = 0.3$ rad/MHz. Hence,

$$\frac{\Delta\phi'}{\Delta\phi} \approx \frac{1}{3}.$$

This reduction in phase shift with feedback was observed at fixed frequencies.

It is extremely important to adjust the lengths of lines to the phase comparator so that, over the frequency range to be used, there will be near zero phase difference at the terminals at the center of the locking range. This is best done with a network analyzer taking the place of the phase comparator. By manually adding a small current to the slave-oscillator coil, one can find a particular phase difference θ_1 at the center of the locking range at some frequency f_1. The phase difference θ_2 is then found at the center of the locking range at another frequency f_2. For a path length difference L between the locked and unlocked sources,

$$\theta_1 = \frac{360}{c} f_1 L$$

$$\theta_2 = \frac{360}{c} f_2 L$$

where c represents velocity of propagation of the lines used, and θ_1 and θ_2 are in degrees. Then,

$$L = \frac{c}{360} \frac{\theta_2 - \theta_1}{f_2 - f_1}.$$

With the correct cable length, the output of the comparator should always change polarity in the same direction when the slave-oscillator free-running frequency becomes higher than the master-oscillator frequency. A similar method is also used to equalize the lengths of signal paths to the final output.

Fig. 4 shows the degree of locking obtained using the above methods. The signal paths to the network analyzer were made slightly unequal in order to decrease the change in phase angle at the low end of the band. For this reason the phase difference at the output is greater than $+90°$ over part of the phase-locked frequency range.

Under the above conditions, the voltage E_Φ was set to incremental values, and a slowly changing ramp current was applied to the series YIG coils. A plot of output phase with frequency (Fig. 5) is shown as a function of the applied voltage E_Φ. As expected, the output phase differences varied linearly with the applied voltage for small phase angles.

It appears that the degree of success, using this method of phase shifting, is very much dependent on the accuracy of frequency tracking. It is expected that varacter-tuned oscillators, in which hysteresis has no effect, would be preferable to the YIG oscillators used in this experiment.

Finally, the feedback method should also correct for differences in phase tracking between similar amplifiers used before the final couplers.

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Notes on the Conjugate Matched Two-Port as an UHF Amplifier

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***Abstract*—Because of the relative simplicity of measurement of scattering parameters of active two-ports at frequencies up to the lower microwave region, investigations have been made into the application of these parameters to the design of UHF amplifiers. The theory of generalized scattering parameters has been developed by Kurokawa [1], applied to two-port power-flow analysis by Bodway [2], and used in the design of a single-stage UHF amplifier by Froehner [3]. In this last paper, the bandwidth limitations imposed by the matching networks were not considered, nor was the capacitive matching arrangement to a purely resistive load fully developed. Both of these topics are the subject of this short paper, in which the relevant design expressions are also given.**

Discussion

In this short paper it will be assumed that the elements S_{ij} of the n-port scattering matrix are well understood and can be used without further definition.

It is shown in the Appendix that for a one-port, for $S_{11} = \Gamma$, its Q can be written

$$Q = \frac{|\Gamma - \Gamma^*|}{1 - |\Gamma|^2} \tag{1}$$

and further, it has been shown [2] that an unconditionally stable two-port can be simultaneously matched at both ports, and under these conditions yields its maximum transducer gain. The source and load reflection coefficients to satisfy this condition were found to be, respectively,

$$\Gamma_{ms} = C_1^* \left\{ \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2|C_1|} \right\} \tag{2}$$

and

$$\Gamma_{ml} = C_2^* \left\{ \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2|C_2|} \right\} \tag{3}$$

in which

$$\begin{aligned} C_1 &= S_{11} - S_{22}^*\Delta \\ C_2 &= S_{22} - S_{11}^*\Delta \\ B_1 &= 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ B_2 &= 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ \Delta &= S_{11}S_{22} - S_{12}S_{21}. \end{aligned} \tag{4}$$

For a narrow-band amplifier, the Q is related to the center frequency ω_0, and the bandwidth $\Delta\omega$ by the well-known relation

$$Q = \frac{\omega_0}{\omega\Delta} \tag{5}$$

and obviously, from (2)–(4), Γ_{ms} and Γ_{ml} are calculable from the measured scattering parameters.

Hence, from these and (1), the maximum bandwidth for this type of load-matching condition is:

$$\Delta\omega = \frac{\omega_0(1 - |\Gamma_{ml}|^2)}{|\Gamma_{ml} - \Gamma_{ml}^*|}. \tag{6}$$

This simple relationship is an obvious aid to design, as it determines the maximum bandwidth available from a given device at a given center frequency (for an unconditionally stable maximum transducer gain).

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If the gain from one stage is insufficient and a multistage amplifier is used, then the bandwidth per stage will be related to the total bandwidth by the well-known relation

$$\Delta\omega_{\text{total}} = \Delta\omega_{\text{per stage}}\sqrt{2^{1/n} - 1} \tag{7}$$

in which n equals the number of stages.

For the second topic of this correspondence, we consider a narrow-band amplifier typically loaded by a resistive load, and the effect of the total output shunt capacitance, which limits the obtainable bandwidth. The total output shunt capacitance C_T is related to the output parallel susceptance across a two-port B by the well-known relation

$$C_T = \frac{1}{2}\frac{\partial B}{\partial\omega} \tag{8}$$

which, for a narrow-band amplifier, reduces to

$$C_T \cong \frac{1}{2}\frac{\Delta B}{\Delta\omega} \tag{9}$$

in which

$$\Delta B \cong B(\omega_0 + \tfrac{1}{2}\Delta\omega) - B(\omega_0 - \tfrac{1}{2}\Delta\omega). \tag{10}$$

A parallel load admittance $Y_l = G_l + jB_l$ can be expressed in terms of the reflection coefficient Γ_{ml} as

$$G_l + jB_l = \frac{1 - |\Gamma_{ml}|^2 + \Gamma_{ml}^* - \Gamma_{ml}}{|1 + \Gamma_{ml}|^2} \tag{11}$$

so that

$$jB_l = \frac{\Gamma_{ml}^* - \Gamma_{ml}}{|1 + \Gamma_{ml}|^2}\,. \tag{12}$$

Hence, from equations (9), (10), and (12)

$$C_T = \frac{1}{2\cdot\Delta\omega}\left|\frac{\Gamma_{ml}^*(\omega_0 + \frac{1}{2}\Delta\omega) - \Gamma_{ml}(\omega_0 + \frac{1}{2}\Delta\omega)}{|1 + \Gamma_{ml}(\omega_0 + \frac{1}{2}\Delta\omega)|^2} - \frac{\Gamma_{ml}^*(\omega_0 - \frac{1}{2}\Delta\omega) - \Gamma_{ml}(\omega_0 - \frac{1}{2}\Delta\omega)}{|1 + \Gamma_{ml}(\omega_0 - \frac{1}{2}\Delta\omega)|^2}\right|. \tag{13}$$

All terms on the right-hand side of (13) are either measurable or stated as a design criterion, and hence for effective constant resistive load matching the total output capacitance C_T, calculated as above for a given bandwidth requirement, must be greater than the active two-port output capacitance C_0.

Appendix

For a general n-port network with scattering matrix $[S]$, the energy dissipated, E_d, and the energy stored, E_s, are given respectively by the real and the imaginary parts of the product $[I_n^*]^T[V_n]$, in which * denotes conjugate and T denotes transpose. If a_k and b_k are the incident and reflected waves at the kth port, using the accepted definition for the n-port, $[b_n] = [S][a_n]$, it can be shown that

$$E_d = [a_n^*]^T[[U] - [S^*]^T[S]][a_n] \tag{14}$$

and

$$E_s = [a_n^*]^T[[S] - [S^*]][a_n] \tag{15}$$

in which $[U]$ is the unit matrix.

We define a matrix $[M]$ by the equation

$$[M] = [U] - [S^*]^T[S] \tag{16}$$

and a matrix $[N]$ by the equation

$$[N] = [S] - [S^*]^T. \tag{17}$$

Thus

$$E_d = [a_n^*]^T[M][a_n] \tag{18}$$

and

$$E_s = [a_n^*]^T[N][a_n]. \tag{19}$$

For time-invariant networks and sinusoidal excitation at the ports, the energy dissipated and the energy stored are both scalar quantities given by:

$$E_d = \sum_{\substack{j=1\\j=1}}^{n,n} a_i^* a_j M_{ij} \tag{20}$$

$$E_s = \sum_{\substack{j=1\\i=1}}^{n,n} a_i^* a_j N_{ij}. \tag{21}$$

And, further, if the incident waves to all ports have the same amplitude and phase, (20) and (21) reduce to:

$$E_d = |a|^2 \sum_{\substack{i=1\\j=1}}^{n,n} M_{ij} \tag{22}$$

$$E_s = |a|^2 \sum_{\substack{i=1\\j=1}}^{n,n} N_{ij}. \tag{23}$$

Hence, if Q for the n-port is defined as the ratio of stored energy to dissipated energy under the conditions of equal amplitude, in-phase incident waves, from (22) and (23),

$$Q_n = \frac{\sum_{\substack{i=1\\j=1}}^{n,n} N_{ij}}{\sum_{\substack{i=1\\j=1}}^{n,n} M_{ij}} \tag{24}$$

which, for a one-port, reduces to

$$Q = \frac{|S_{11} - S_{11}^*|}{1 - |S_{11}|^2}\,. \tag{25}$$

For the usual case, where the reference impedance is equal to the driving-point impedance, $S_{11} = \Gamma$, and

$$Q = \frac{|\Gamma - \Gamma^*|}{1 - |\Gamma|^2}$$

given earlier as (1).

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A Multioctave Microstrip 50-Ω Termination

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***Abstract*—A low-cost microstrip 50-Ω termination is described having a maximum VSWR of 1.46 (including the mismatch contributed by a microstrip launcher) from 1 to 18 GHz. It consists of a thin-film chip resistor with a matching structure. Experimental resistor characterization, equivalent-circuit modeling, and matching considerations are presented.**

This short paper describes an integrated-circuit 50-Ω termination, fabricated on 0.025-in ceramic microstrip, that operates from 1 to 18 GHz with good performance.

The termination is illustrated in Fig. 1. It consists of a thin-film

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